DOCUMENT RESUME

ED 252 434 SE 045 368

AUTHOR Cormier, Pierre A.; And Others
TITLE A General Model for Simple and Complex Addition.
PUB DATE [84]
NOTE 14p.; Paper presented at the Annual Meeting of the American Psychological Association (92nd, Toronto, Canada, August, 1984).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Addition; *College Students; Educational Research; Higher Education; *Learning; *Mathematical Models; *Mathematics Education; *Mathematics Instruction; Speeches
IDENTIFIERS *Mathematics Education Research

ABSTRACT

A sample of 24 subjects was presented with 800 addition problems to test a general model of simple and complex addition. According to this model, an addition problem is encoded and verified by column, with specific addition strategies determined by the size of the problem. Two hundred problems each of the following four types were presented: (1) two single-digit addends (type I); (2) one single- and one double-digit addend (type II); (3) two double-digit addends (type III); and (4) three single-digit addends (type IV). Multiple regression analyses applied to average reaction time data identified the specific strategies for each type of problem, with multiple r's ranging between .80 and .90, and supported a self-terminating strategy for complex problems. A discussion of the general model is offered.

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED252434

A GENERAL MODEL FOR SIMPLE AND COMPLEX ADDITION

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
✓ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

PIERRE A. CORMIER, KEITH F. WIDAMAN, & DAVID C. GEARY
UNIVERSITY OF CALIFORNIA AT RIVERSIDE

PAPER PRESENTED AT THE 92ND ANNUAL MEETING OF THE AMERICAL PSYCHOLOGICAL ASSOCIATION, TORONTO, CANADA, AUGUST 1984

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Pierre A. Cormier
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 045 368

ABSTRACT

A SAMPLE OF 24 SUBJECTS ~~WERE~~ was PRESENTED WITH 800 ADDITION PROBLEMS TO TEST A GENERAL MODEL OF SIMPLE AND COMPLEX ADDITION. ACCORDING TO THIS MODEL, AN ADDITION PROBLEM IS ENCODED AND VERIFIED BY COLUMN, WITH SPECIFIC ADDITION STRATGIES DETERMINED BY THE SIZE OF THE PROBLEM. TWO HUNDRED PROBLEMS EACH OF THE FOLLOWING FOUR TYPES WERE PRESENTED: 1) TWO SINGLE-DIGIT ADDENDS (TYPE I); 2) ONE SINGLE- AND ONE DOUBLE-DIGIT ADDEND (TYPE II); TWO DOUBLE-DIGIT ADDENDS (TYPE III); AND THREE SINGLE-DIGIT ADDENDS (TYPE IV). MULTIPLE REGRESSION ANALYSES APPLIED TO AVERAGE REACTION TIME DATA IDENTIFIED THE SPECIFIC STRATIGIES FOR EACH TYPE OF PROBLEM, WITH MULTIPLE R'S RANGING BETWEEN .80 AND .90, AND SUPPORTED A SELF-TERMINATING STRATEGY FOR COMPLEX PROBLEMS. A DISCUSSION OF THE GENERAL MODEL IS OFFERED.

A GENERAL MODEL FOR SIMPLE AND COMPLEX ADDITION

CHRONOMETRIC TECHNIQUES HAVE BEEN SUCCESSFULLY APPLIED TO THE STUDY OF MENTAL ADDITION (ASHCRAFT, 1982). PREVIOUS RESEARCH HAS SHOWN THAT BOTH COUNTING BASED MODELS AND DIRECT MEMORY RETRIVAL MODELS HAVE BEEN USED TO REPRESENT COGNITIVE PROCESSES UNDERLYING SIMPLE (PARKMAN & GROEN, 1971; ASHCRAFT & BATTAGLIA, 1978) AND COMPLEX (ASHCRAFT & STAZYK, 1981) ADDITION. HOWEVER, THIS RESEARCH HAS BEEN LIMITED BY THE LACK OF COMPLETENESS AND REPRESENTATIVENESS OF THE COMPLEX ARITHMETIC PROBLEMS STUDIED. THE PRESENT STUDY SOUGHT TO EXTEND PREVIOUS RESEARCH BY PROPOSING AND TESTING A GENERAL MODEL FOR BOTH SIMPLE AND COMPLEX ADDITION ON A MORE REPRESENTATIVE ARRAY OF ADDITION PROBLEMS.

THE PROPOSED GENERAL MODEL, SHOWN IN FIGURE 1, IS MULTI-STAGED. THE FIRST TWO PROCESSES INVOLVE ENCODING THE TYPE OF PROBLEM, AND THEN ENCODING THE ADDENDS (COLUMNWISE) BEGINNING WITH THE UNITS PLACE. THE UNITS ADDENDS ARE SUMMED, EITHER THROUGH RETRIEVAL FROM A NETWORK OF STORED FACTS OR PROCEDURALLY. THE OBTAINED UNIT SUM IS THEN COMPARED TO THE STATED UNIT SUM, AND THEN A DECISION AS TO THE CORRECTNESS IS MADE. IF THE UNIT SUM IS INCORRECT THE PROBLEM IS TERMINATED AND A RESPONSE (NO) IS GIVEN. IF THE UNIT SUM IS CORRECT THE PROCESS "LOOPS BACK". IF THE SUM OF THE COLUMN IS GREATER THEN 10, A CARRY PROCESS IS INVOKED AND A ONE IS INCREMENTED IN THE SUCCEEDING (TENS) COLUMN. THE ADDENDS FOR THE TENS COLUMN ARE ENCODED, AND THEN SUMMED, ECT. THE LOOPING IS CONTINUED UNTIL ALL COLUMNS ARE COMPLETED OR UNTIL ONE COLUMN IS INCORRECT, AT WHICH POINT THE PROBLEM IS SELF-TERMINATED.

METHODS

SUBJECTS

SUBJECTS WERE 24 UNDERGRADUATE STUDENTS ENROLLED IN INTRODUCTORY PSYCHOLOGY CLASSES AT THE UNIVERSITY OF CALIFORNIA AT RIVERSIDE. THERE WERE 9 MALES AND 15 FEMALES RANGING IN AGE FROM 16 TO 29 YEARS (X=19.6). NINETEEN SUBJECTS WERE RIGHT HANDED AND FIVE SUBJECTS WERE LEFT HANDED.

TASK

A TOTAL OF 800 ADDITION PROBLEMS WERE PRESENTED TO EACH SUBJECT ACROSS TWO ONE-HOUR SESSIONS. THIS GLOBAL SET COMPRISED 200 PROBLEMS OF EACH OF FOUR TYPES OF ADDITION.

TYPE I: TWO SINGLE DIGIT ADDENDS (EX: 3 + 2)

TYPE II: ONE SINGLE DIGIT AND ONE DOUBLE DIGIT ADDEN (EX: 43 + 2)

TYPE III: TWO DOUBLE DIGIT ADDENDS (EX: 43 + 56)

TYPE IV: THREE SINGLE DIGIT ADDENDS (EX: 2 + 8 + 6)

FOR EACH PROBLEM TYPE, THERE WERE 100 CORRECT AND 100 INCORRECT PROBLEMS. FOR TYPES I AND IV, INCORRECT SUMS WERE OFF BY + OR - 1 FOR 50 PROBLEMS, AND + OR - 2 FOR THE REMAINING 50 PROBLEMS. FOR TYPES II AND III, INCORRECT SUMS WERE OFF BY + OR - 1 FOR 25 PROBLEMS, BY + OR - 2 FOR 25 PROBLEMS AND BY + OR - 10 FOR THE REMAINING 50 PROBLEMS. PROBLEMS WERE DIVIDED INTO TWO RANDOMLY ORDERED SET OF 400 PROBLEMS. EACH OF THESE SETS WAS DESIGNED AS 100 QUATERNIONS OF RANDOMLY ORDERED PROBLEMS WITH ONLY ONE PROBLEM OF EACH TYPE OF ADDITION WITHIN EACH QUARTERNION AND ITS POSITION WITHIN THE QUARTERNION WAS RANDOM. ORDER OF ADMINISTRATION OF PROBLEM SETS WAS COUNTERBALANCED ACROSS SUBJECTS. EACH PROBLEM WAS PRESENTED ON A VIDEO SCREEN IN COLUMN FORM, AND THE SUBJECT PRESSED ONE RESPONSE KEY TO INDICATE THE PRESENTED SUM WAS CORRECT OR A SECOND KEY TO INDICATE THE SUM WAS INCORRECT.

RESULTS AND DISCUSSION

BASED ON GENERAL STAGE PROCESSING MODELS, MODELS FOR ADDITION WERE FIT TO AVERAGE REACTION TIME (RT) DATA USING MULTIPLE REGRESSION TECHNIQUES. TABLE I PRESENTS ANALYSES ON TYPE 1 PROBLEMS FOR THE FIVE COUNTING BASED MODELS OF PARKMAN & GROEN (1971) AND THE MEMORY RETRIEVAL NETWORK MODEL OF ASHCRAFT (1982). RESULTS PRESENTED IN TABLE 1 ARE CONSISTENT WITH PREVIOUS FINDINGS. ASHCRAFT'S (1982) MEMORY RETRIEVAL MODEL (REPRESENTED BY TRUESUM SQUARED) EXPLAINED 66 PER CENT OF THE RT VARIANCE. THE TRUTH VALUE REPRESENTS INTERCEPT DIFFERENCES (IN MSEC.) BETWEEN TRUE AND FALSE PROBLEMS WITH CORRECT PROBLEMS BEING PROCESSED 119 MSEC. FASTER THEN INCORRECT PROBLEMS. THIS PARAMETER ESTIMATE, AS WELL AS THE REMAINING PARAMETER ESTIMATES, ARE COMPARABLE WITH PREVIOUS RESEARCH. PARKMAN AND GROEN'S (1971) MIN MODEL PROVIDED THE BEST COUNTING BASED REPRESENTATION OF THE RT DATA, AND EXPLAINED 65 PER CENT OF THE RT VARIANCE. IN THIS MODEL A MENTAL COUNTER IS SET TO THE LARGER (MAX) OF THE TWO ADDENDS AND THE SMALLER (MIN) ADDEND IS INCREMENTED BY ONES UNTIL A SUM IS OBTAINED.

TABLE 2 PRESENTS THE FOUR BEST MODELS EXPLAINING RT DATA FOR TYPE II PROBLEMS. INSPECTION OF TABLE 2 REVEALS A "COUNTING ALL" STRATEGY BEST EXPLAINED RT DATA. SPECIFICALLY, THE UNITSUM IN THE ONE'S COLUMN AND THE MAXIMUM (LARGER) ADDEND IN THE TEN'S COLUMN WERE THE BEST COLUMNWISE PREDICTORS OF THE FIVE COUNTING BASED AND THE MEMORY RETRIEVAL MODELS. THE PROPOSED SELF-TERMINATING PROCESS, IN WHICH IT WAS EXPECTED THAT ONCE AN INCORRECT SUM (E.G. IN THE ONE'S COLUMN) WAS ENCOUNTERED THE SUBJECT WOULD "EXIT" THE PROBLEM AND RESPOND ("NO"), IS SUPPORTED. THIS IS REFLECTED IN A COMPARISON OF THE TWO "COUNTING ALL" MODELS AND THE CARRYEXH (INCLUDES A CARRY PROCESS AND IS EXHAUSTIVE: I.E. EVEN IF THE ONE'S SUM IS INCORRECT THE SUBJECT CONTINUES TO THE 10'S COLUMN AND COMPLETES THE PROBLEM), CARRYST (I.E. A CARRY AND SELF-TERMINATING PARAMETER: IF THE UNITS SUM IS INCORRECT NO CARRY IS MADE AND THE PROBLEM

IS IMMEDIATELY TERMINATED AND A "NO" RESPONSE IS EXECUTED), AND NCOMP (NUMBER OF COMPARISONS TO BE MADE: I.E. 1 OR 2 FOR THE UNIT SUM AND THE TEN SUM). THESE SELF-TERMINATING PARAMETERS EXPLAINED AN ADDED 8 PER CENT OF THE RT VARIANCE.

ANALYSES ON TYPE III PROBLEMS ARE PRESENTED IN TABLE 3. INSPECTION OF TABLE 3 REVEALS THAT TWO ASHCRAFT BASED MODELS AND A "COUNTING ALL" MODEL PROVIDED EQUALLY POWERFUL REPRESENTATIONS OF RT DATA: ALL EXPLAINED NEARLY 82 PER CENT OF THE RT VARIANCE. ALL THREE MODELS CLEARLY SUPPORTED PARAMETERS REPRESENTING CARRY AND SELF-TERMINATING PROCESSES, AS WELL AS RT DIFFERENCES BETWEEN TRUE AND FALSE PROBLEMS (TRUTH) AND NUMBER OF COMPARISONS (NCOMP). THE PROCESSES BY WHICH THE SUM IS OBTAINED IS UNCLEAR. FOR THESE DATA, PARAMETERS REPRESENTING RT DATA AS A LINEAR FUNCTION OF UNITSUM AND TENSUM, OR AS A QUARDRATIC FUNCTION OF UNITSUM AND TENSUM (E.G. UNITSUMSQ), OR AS A SUM OF THE TWO COLUMN SUMS SQUARED (UNITSUMSQ + TENSUMSQ: ASHCRAFTEX) CANNOT BE DIFFERENTIATED. REGARDLESS, A COLUMNWISE ADDITION OF DOUBLE-DIGIT ADDENDS WITH CARRY AND SELF-TERMINATING PROCESSES ARE CLEARLY SUPPORTED.

TABLE 4 PRESENTS THE FIVE MOST EXPLANATORY MODELS FOR TYPE IV PROBLEMS. INSPECTION OF TABLE 4 REVEALS THAT, AGAIN, A COUNTING BASED MODEL AND AN ASHCRAFT BASED MODEL EXPLAIN NEARLY EQUIVALENT AMOUNTS OF RT VARIANCE (ABOUT 81 PER CENT). THE COUNTING BASED MODEL REPRESENTS RT DATA AS A LINEAR FUNCTION OF THE SUM OF THE TWO LARGEST ADDENDS FOR THE FIRST PARAMETER, AND A LINEAR FUNCTION OF THE SMALLEST ADDEND AS THE SECOND PARAMETER. AS WITH OTHER MODELS AND PREVIOUS RESEARCH, TRUE PROBLEMS ARE PROCESSED FASTER THAN INCORRECT PROBLEMS. THE ASHCRAFT BASED MODEL DIFFERS FROM THE COUNTING BASED MODEL ONLY IN THAT THE SUM OF THE TWO LARGER ADDENDS IS REPRESENTED AS A QUADRATIC FUNCTION OF RT. THIS WOULD REPRESENT A MEMORY RETRIEVAL STRATEGY. ESTIMATES FOR THE TRUTH AND SMALLEST PARAMETERS ACROSS ALL FIVE MODELS ARE EQUIVALENT.

RESULTS FOR ALL FOUR PROBLEM TYPES, BUT ESPECIALLY FOR THE MORE COMPLEX

PROBLEMS, PROVIDE SUPPORT FOR THE PROPOSED GENERAL MODEL. ADDITION PROBLEMS SEEM TO BE DONE COLUMNWISE WITH SPECIFIC ADDITION STRATEGIES DETERMINED BY THE SIZE OF THE PROBLEM. ANALYSES OF COMPLEX PROBLEMS CLEARLY SUPPORT A CARRY PROCESS, AND A SELF-TERMINATING PROCESS, IN WHICH THE SUBJECT TERMINATES AND EXITS THE PROBLEM ONCE AN INCORRECT SUM IS ENCOUNTERED. FUTURE RESEARCH WILL EXPLORE THE APPLICABILITY OF THIS GENERAL MODEL FOR MORE COMPLEX PROBLEMS.

FIGURE 1

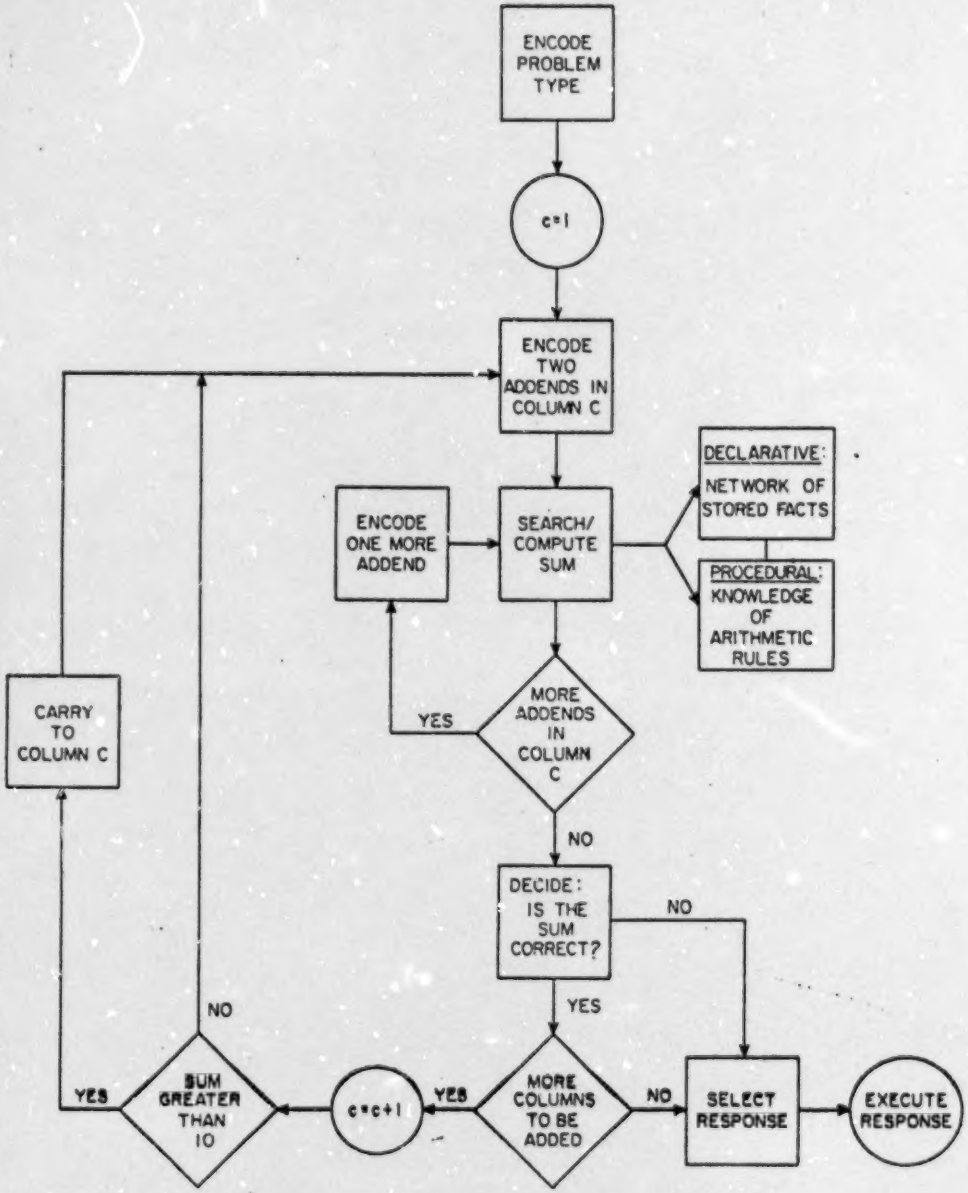
ENCODE PROBLEM TYPE
c=1
ENCODE TWO ADDENDS IN COLUMN C
ENCODE ONE MORE ADDEND
SEARCH/ COMPUTE SUM
DECLARATIVE: NETWORK OF STORED FACTS
PROCEDURAL: KNOWLEDGE OF ARITHMETIC RULES
MORE ADDENDS IN COLUMN C
YES
NO
CARRY TO COLUMN C
DECIDE: IS THE SUM CORRECT?
NO
YES
SUM GREATER THAN 10
NO
YES
c=c+1
YES
MORE COLUMNS TO BE ADDED
NO
SELECT RESPONSE
EXECUTE RESPONSE

Table 1

Parameter-, F-, and R-SQUARE values associated with the six models used to predict reaction time on TYPE I problems (i.e two single-digit addends)

Model	Variable	Regression weight		Full Model F-value	R-SQUARE
Parkman & Groen	ADDEND2	32.82			
	TRUTH	116.56			
	Interaction	15.07	n.s.		
	Intercept	938.37		38.05	.393430
Parkman & Groen	ADDEND1	31.16			
	TRUTH	148.83			
	Interaction	7.90	n.s.		
	Intercept	945.87		28.90	.330059
Parkman & Groen	MAX	44.80			
	TRUTH	92.32	n.s.		
	Interaction	14.53	n.s.		
	Intercept	802.31		36.97	.386588
Parkman & Groen	TRUSUM	35.99			
	TRUTH	68.12	n.s.		
	Interaction	12.92			
	Intercept	762.18		97.40	.624081
Parkman & Groen	MIN	63.16			
	TRUTH	119.79			
	Interaction	24.21			
	Intercept	917.65		108.12	.648258
Ashcraft	(TRUSUM)2	2.04			
	TRUTH	119.20			
	Interaction	0.68			
	Intercept	890.60		115.41	.662979

*In all cases, nontie problems and DF = 3,176.

Note: The regression weights indicate the amount of time in milliseconds (msec.) it takes to perform the operation described by the regression model. For example, in the first model on this table, the subject sets her mental counter to the first addend she reads (ADDEND1) and then increment that value by one a number of times equal to the second addend in the problem (ADDEND2). Each incrementing step takes approximately 33 msec. Problems with an invalid sum take an additionnal 117 msec. to process, as is indicated by the regression weight of the TRUTH variable. All the other models in the three tables can be interpreted similarly.

Table 2

Parameter-, F-, and R-SQUARE values associated with general models accounting for most of the variance in RT for TYPE II problems (i.e. one single-digit + one double-digit addend)

Model	Variable	Regression weight	P	Full Model F-value	R-SQUARE
Parkman & Groen	UNITMIN	107.54	0.0001		
	CARRYEXT	253.60	0.0001		
	TENMAX	21.04	0.0190		
	TRUTH	232.50	0.0038		
	UNITMIN*TRUTH	-36.81	0.0096		
	TENMAX*TRUTH	-11.59	0.3573		
	Intercept	1139.69	0.0001	65.18	.669580
Parkman & Groen	UNITMIN	110.32	0.0001		
	CARRYST	235.94	0.0001		
	TENMAX	21.28	0.0079		
	NCOMP	216.73	0.0001		
	TRUTH	338.52	0.0001		
	UNITMIN*TRUTH	-26.58	0.0422		
	TENMAX*TRUTH	-6.86	0.5404		
	Intercept	705.13	0.0001	77.87	.739510
Counting all	UNITSUM	60.86	0.0001		
	CARRYEXT	193.12	0.0016		
	TENMAXC	23.85	0.0105		
	TRUTH	322.07	0.0013		
	UNITSUM*TRUTH	-19.51	0.0210		
	TENMAXC*TRUTH	-14.01	0.2777		
	Intercept	898.04	0.0001	56.76	.638273
Counting all	UNITSUM	61.96	0.0001		
	CARRYST	181.57	0.0006		
	TENMAXC	24.14	0.0033		
	NCOMP	237.75	0.0001		
	TRUTH	405.78	0.0001		
	UNITSUM*TRUTH	-13.41	0.0871		
	TENMAXC*TRUTH	-9.97	0.3820		
	Intercept	416.13	0.0013	68.49	.721098

Table 3

Parameter-, F-, and R-SQUARE values associated with models accounting for variance in RT for TYPE III problems (two double-digit addends)

Model	Variable	Regression weight	P	Full Model F-value	R-SQUARE
Parkman & Groen	MIN	9.15	0.0001		
	TRUTH	-259.95	0.0004		
	Intercept	2042.41	0.0001	20.42	.171706
Ashcraft	TRUSUMSQ	0.03	0.0001		
	TRUTH	-259.95	0.0003		
	Intercept	2066.64	0.0001	23.28	.191152
Counting all	UNITSUM	42.06	0.0001		
	CARRYST	361.00	0.0001		
	TENSUM	55.92	0.0001		
	NCOMP	186.01	0.0197		
	TRUTH	187.55	0.0001		
	Intercept	895.25	0.0001	173.08	.816875
Ashcraft	UNITSUMSQ	2.34	0.0001		
	CARRYST	311.01	0.0001		
	TENSUMSQ	2.54	0.0001		
	NCOMP	464.41	0.0001		
	TRUTH	195.89	0.0001		
	Intercept	766.53	0.0001	174.17	.817811
Ashcraft	ASHCEXT	2.47	0.0001		
	CARRYST	299.73	0.0001		
	NCOMP	478.97	0.0001		
	TRUTH	196.10	0.0001		
	Intercept	738.86	0.0001	218.55	.817621

*In all cases, non of the interactions were significant.

Table 4

Parameter-, F-, and R-SQUARE values associated with the most important models used to predict reaction time on TYPE IV problems (i.e. three single-digit addends)

Model	Variable	Regression weight	Full Model F-value	R-SQUARE
Parkman & Groen	TWOSMALL	149.99		
	LARGEST	21.41 n.s.		
	TRUTH	160.13		
	Intercept	841.96	223.30	.773647
Ashcraft	(TWOSMALL)2	9.23		
	LARGEST	65.02		
	TRUTH	161.78		
	Intercept	992.21	157.55	.706866
Parkman & Groen	TWOLARGE	53.18		
	SMALLEST	245.99		
	TRUTH	163.33		
	Intercept	863.85	280.99	.811351
Ashcraft	(TWOLARGE)2	2.39		
	SMALLEST	246.18		
	TRUTH	162.90		
	Intercept	1129.10	272.09	.806378
Parkman & Groen	MIN2	87.20		
	SMALLEST	243.70		
	TRUTH	162.10		
	Intercept	1097.57	266.66	.803206

*In all cases, none of the interactions were significant and DF = 3,196. All parameters are significant at the .0001 level.